Announcements

1) HW 5 up sometime tomorrow

2) Advising Sessions

Today 3-4 & tomorrow 3-4
math Library

3) Today 4:10 Math Club talk

Recall. A set $S \subseteq X$ (X a metric space with metric d) is **open** if $\forall x \in S$, $\exists \varepsilon > 0$ with $B(x, \varepsilon) \subseteq S$.

Proposition: (properties of open sets)

Let X be a metric space and I an index set of any cardinality. Then

1) If $\{S_i\}_{i \in I}$ is a collection of open sets in X, then $\boxed{\bigcup_{i \in I} S_i}$ is open in X.

2) If $S_1, S_2, \ldots, S_n$ are open in X, then $\bigcap_{i=1}^{n} S_i$ is open in X.

Proof:

1) Let $x \in \bigcup_{i \in I} S_i$.

Then there is a $j \in I$, $x \in S_j$. Since S_j is open, $\exists\ \varepsilon > 0$ with

$$B(x, \varepsilon) \subseteq S_j \subseteq \bigcup_{i \in I} S_i$$

2) Let $x \in \bigcap_{i=1}^{n} S_i$.

Since $x \in S_i\ \forall\ 1 \leq i \leq n$, $\exists$ $\varepsilon_i > 0$ with $B(x, \varepsilon_i) \subseteq S_i$ (each S_i is open)

Let $\varepsilon = \min\{\varepsilon_i \mid 1 \leq i \leq n\}$

and observe that $\varepsilon > 0$!

Then $\forall\, i,\ 1 \leq i \leq n,$

$$B(x, \varepsilon) \subseteq B(x, \varepsilon_i) \ (\varepsilon \leq \varepsilon_i)$$

$$\subseteq S_i,$$ which

implies $$B(x, \varepsilon) \subseteq \bigcap_{i=1}^{n} S_i$$

$\square$

Definition: (limit point)

Let $S \subseteq X$, X a metric space. A point $x \in X$ (not necessarily in S) is called a limit point of S if $\forall\ \varepsilon > 0$, $B(x, \varepsilon)$ intersects S in a point other than x.

Example 1: ($\mathbb{R}$)

Let $S = (0,1)$.

The points $x=0$ and $x=1$ are limit points of S even though $0,1 \notin S$

This is because $\forall\, \varepsilon > 0$, $B(0,\varepsilon) = (-\varepsilon, \varepsilon)$, and

$$(-\varepsilon,\varepsilon) \cap (0,1) = \begin{cases} (0,1), & \varepsilon \geq 1 \\ (0,\varepsilon), & \varepsilon < 1 \end{cases}$$

similar for $x=1$.

Definition: (closed set)

A subset S of a metric space X is **closed** if it contains all of its limit points